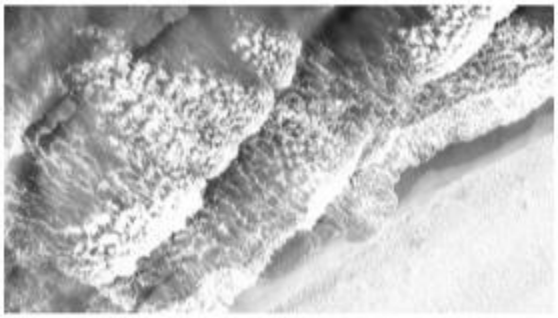
Waves

Case Study Based Questions

Read the following passages and answer the questions that follow:

1. Water waves near the shore give the appearance of being transverse waves. Actually, are more closely by a combination of transverse and longitudinal waves. This becomes apparent when one watches a small Floating in the water-it not only bobs up and down but also move to-and-fro. The frequencies and wavelengths of these two simultaneous, but mutually perpendicular, waves are the same but their amplitudes, of course, need not be the same.



(A) If the observed to crest distance of the travelling water waves is 3.0 m and the speed of the waves toward the shore is 2.0 m/s, The frequency of the waves will be:

(a) 0.667 Hz

(b) 0.867 Hz

(c) 0.650 Hz

(d) 0.600 Hz

(B) The equation of spherical progressive wave is:

(a) $y = A \sin (kx - \omega t)$

(b) $y = \frac{A}{r} \sin (kx - \omega t)$

(c) $y = \frac{A}{\sqrt{r}} \sin (kx - \omega t)$

(d) $y = \frac{A}{2r} \sin (kx - \omega t)$

(C) If the speed of longitudinal mechanical waves in water is 1400 m/s, then the Bulk modulus of elasticity of water is. (Density of water 1 g/cm^3)

(a) 1.96 x 109 N/m^2
(b) 2.08 x 109 N/m^2
(c) 5.64 x 109 N/m^2
(d) 1.02 x 109 N/m^2

(D) The velocity of a pulse in a rope of mass/length, μ = 3.0 kg/m and the tension is 25 N is:

(a) 2.00 m/s
(c) 2.89 m/s
(b) 2.59 m/s
(d) 3.12 m/s

(E) A transverse wave in a cord of length is L = 3.0 m and mass M = 12.0 g travelling at 6000 cm/s. The tension in the cord is:

(a) 14.4 N
(b) 15.2 N
(c) 15.8 N
(d) 16.3 N

Ans. (A) (a) 0.667 Hz

Explanation: The distance of the travelling water waves, λ = 3.0 m And the speed of waves v = 2.0 m/s As we know that,

frequency, $f = \frac{v}{\lambda}$

The frequency of the wave $f = 0.667$ Hz

(B)

(b) $y = \frac{A}{r} \sin(kx - \omega t)$

Explanation: $I = \frac{P}{A} = \frac{P}{4\pi r^2}$

Amplitude is given as,

$$A = \sqrt{I} = \sqrt{\frac{P}{4\pi r^2}}$$

$$A \propto \frac{1}{r}.$$

(C) (a)1.96 x 10^9 N/m^2

Explanation: Speed of longitudinal wave is,

$$v = \sqrt{\frac{B}{\rho}}$$

$$140000 \text{ cm/s} = \sqrt{\frac{B}{1}}$$

$$\Rightarrow \quad B = 1.96 \times 10^{10} \text{ dyne/cm}^2$$
$$= 1.96 \times 10^{9} \text{ N/m}^2$$

(D) (c) 2.89 m/s

Explanation: For transverse waves in a cord, the velocity of propagation is given by,

$$v_P = (S)\ \frac{1}{2}$$

$$= [25 \text{ N} \times 3.0 \text{ kg/m}]\ \frac{1}{2}$$

$$= 2.89 \text{ m/s}$$

(E) (a) 14.4 N

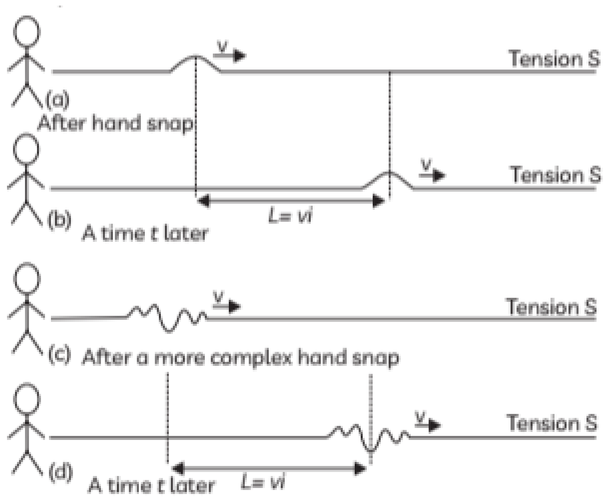
Explanation: For transverse waves in a cord, the velocity of propagation is given by.

$$v_P = (S)\frac{1}{2}$$

So, by the equation,

$$S = \mu v_{p^2} = MLv_{p^2}$$
$$= [0.012 \text{ kg} \times 3.0 \text{ m}]\ 60 \text{ m/s})^2$$
$$= 14.4 \text{ N}$$

2. Consider a student holding one end of a very long cord under tension S, with the far end attached to a wall. If the student suddenly snaps her hand upward and back down, while keeping cord under tension, a pulse, something like that shown in figure (a) will appear to rapidly travel along the cord away from the student. If the amplitude of the pulse (its maximum vertical displacement) is not large compared to its length, the pulse will travel at a constant speed, v, until it reaches the tied end of the cord. In general, shape of the pulse remains the same as it travels in figure (b)], and its size diminishes only slightly (due to thermal losses) as it propagates along the cord. By rapidly shaking her hand in different ways, the student can have pulses of different shapes [e.g., in figure (c)] travelling down the cord. As long as the tension S, in the cord is the same for each such snap, and the amplitudes are not large, the speed of all the pulses in the cord will be the same no matter what their shapes.

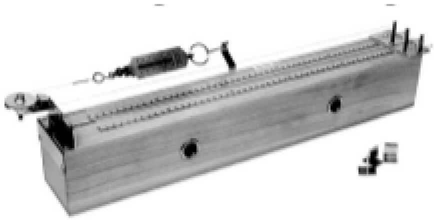


(A) For the cases of the figure, in what direction are the molecules of the cord moving as they pass by?
(B) If actual molecules of cord are not travelling the pulse, what is the K.E.?
(C) What qualitative explanation can you give for this phenomenon?

Ans. (A) We can understand the motion of the cord molecules as the pulse approaches a point in the cord and passes by. First, the molecules at a given horizontal point on the cord move upward, until the maximum of the pulse passes the point, at which the molecules are at the maximum vertical displacement (the amplitude), then the molecules move back down until they return to their normal position as the pulse passes by. Thus, the molecules move perpendicular to the direction in which the pulse moves.
(B) The shape of the pulse travels as one set of molecules after another goes through the vertical motion described in part (A). The pulse carries energy, the vertical kinetic energy of the moving molecules and the associated potential energy due to the momentary stretching of the cord, in the pulse region.
(C) As the tension in the cord, is increased forces between adjacent molecules get stronger, resisting the effort to pull the cord apart. When the student snaps the end of the cord upward the adjacent molecules are forced upward as well and so are the next of molecules and so on. All the molecules in the cord don't move upward at the same instant, however, it takes some time for each succeeding set of molecules to feel the resultant force caused by the slight motion of the prior set away from them. While the successive groups of molecules are being pulled upward, the student snaps her hand back down, so the earlier molecules are reversing direction and moving back down. The net effect is that successive sets of molecules down the length of the cord start moving

upward while further back other sets are feeling the pull back down. This process the pulse to, in effect, reproduces itself over and over again down the cord.

3. A Sonometer is defined as the device that is used for demonstrating the relationship between the frequency of the sound that is produced by the string when it is plucked and the tension, length, and mass per unit length of the string. The sound is produced in the transverse standing wave in the string.



(A) A Sonometer wire is vibrating in resonance with a tuning fork still be is resonance with the wire?

(B) Why do tuning forks have two prongs?

(C) Hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is 1.7 kms^{-1}? The operating frequency of the scanner is 4.2 MHz.

Ans. (A) When a wire of length L vibrates its resonant frequency in nth mode after stretching it by a tension T, then frequency

of n^{th} harmonic is, $v = \frac{n}{2L}\sqrt{\frac{T}{m}}$, here m is

mass per unit length of stretched wire. Let in given two cases,

$$v_1 = \frac{n}{2L_1}\sqrt{\frac{T_1}{m_1}}$$

$$v_2 = \frac{n}{2L_2}\sqrt{\frac{T_2}{m_2}}$$

In given question,

$$T_1 = T_2 = T$$

$$m_1 = m_2 = m$$

as wire same $\quad L_2 = 2L_1$

$$\frac{v_1}{v_2} = \frac{n_1\sqrt{T}\sqrt{m} \times 2 \times 2L_1}{n_2\sqrt{T}\sqrt{m}\,2L_1}$$

$$= \frac{2n_1}{n_2}$$

As the tuning fork is the same, i.e., in both harmonics n_1 and n_2 frequency of resonance same,

$\therefore \quad v_1 = v_2$

or $\quad \frac{2n_1}{n_2} = 1$

$$n_2 = 2n_1$$

(B) The two prongs of a tuning fork set each other is resonant vibrations and help to maintain the vibrations for a longer time.

(C) Speed of sound in the tissue,

v = 1.7 km/s

$= 1.7 \times 10^3$ m/s

Operating frequency of the scanner,

ν = 4.2 MHz

$= 4.2 \times 10^6$ Hz

The wavelength of sound in the tissue is given as:

$$\lambda = \frac{v}{\nu}$$

$$\lambda = \frac{1.7 \times 10^3}{4.2 \times 10^5}$$

$$\lambda = 4.1 \times 10^{-4} \text{ m}$$

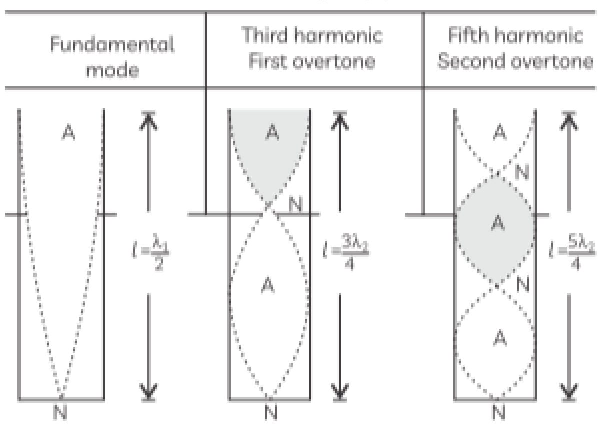
4. Organ pipes are the musical instruments which are used for producing musical sound by blowing air into the pipe. Longitudinal stationary waves are formed on account of superimposition of incident and reflected longitudinal waves.

Equation of standing wave,

$$y = 2a\cos\frac{2\pi t}{\lambda}\sin\frac{2\pi x}{\lambda}$$

Frequency or vibration, $n = \frac{\nu}{\lambda}$

Closed organ pipe



(A) Assertion (A): When we start filling an empty bucket with water, the pitch of sound produced goes on decreasing.

Reason (R): The frequency of man voice is usually higher than that of woman.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

(B) A closed pipe and an open pipe have their first overtones identical in frequency. Their lengths are in the ratio of:

(a) 1:2

(b) 2:3

(c) 3:4

(d) 4:5

(C)The first overtone in a closed pipe has a frequency:

(a) Same as the fundamental frequency of an open tube of same length.

(b) Twice the fundamental frequency of an open tube of same length.

(c) Same as that of the first overtone of an open tube of same length.

(d) None of the above.

(D) An empty vessel is partially filled with water, then the frequency of vibration of air column in the vessel:

(a) Remains the same

(b) Decreases

(c) Increases

(d) First increases, then decrease

(E) It is desired to increase the fundamental resonance frequency in a tube which is closed at one end. This can be achieved by:

(a) Replacing the air in the tube by hydrogen gas.

(b) Increasing the length of the tube.

(c) Decreasing the length of the tube.

(d) All of the above.

Ans. (A) (d) A is false and R is also false.

Explanation: A bucket can be treated as a pipe closed at one end. The frequency of the note produced $L = \frac{v}{4L}$, here L equal to

depth of water level from the open end. As the bucket is filled with water L decreases, hence frequency increases. Therefore, frequency or pitch of sound produced goes on increasing. Also, the frequency of woman voice is usually higher than that of man.double of the closed pipe.

(B) (c) 3:4

It is given that First overtone of closed pipe = First overtone of open pipe

$$\Rightarrow \quad 3\left(\frac{v}{4l_1}\right) = 2\left(\frac{v}{2l_2}\right);$$

where 1 and 2 are the lengths of closed and open organ pipes.

Hence, $\frac{l_1}{l_2} = \frac{3}{4}$

(C) (d) None of the above.

Explanation: First overtone for closes pipe

$$= \frac{3v}{4l}$$

Fundamental frequency for open pipe

$$= \frac{v}{2l}$$

First overtone for open pipe

$$= \frac{2v}{2l}.$$

(D) (c) Increases

Explanation: For closed pipe in general,

$$n = \frac{v}{4l}(2N-1)$$

$$\Rightarrow \quad n \propto \frac{1}{l}$$

i.e., if length of air column decreases frequency increases.

(E) (d) All of the above

Explanation: Fundamental frequency for closed pipe

$$n = \frac{v}{4l}$$

where, $v = \sqrt{\frac{\gamma RT}{M}}$

$$\Rightarrow \quad v \propto \frac{1}{\sqrt{M}}$$

$$\therefore \quad M_{H_2} < M_{air}$$

$$\Rightarrow \quad v_{H_2} < v_{air}$$

Hence, fundamental frequency with H will be more as compared to air.

Also $n \propto \frac{1}{l}$, hence if l decrease n increases

It is well known that (n) = 2(n), hence alloptions are correct.